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13. ABSTRACT *(Maximum 200 words)*
An experimental technique is described for mapping the wavenumber spectrum of the normal mode field as a function of position in a shallow water waveguide with three-dimensional variation in its acoustic properties. These modal maps provide a characterization of the modal properties of the waveguide, can be used as input data to inversion techniques for inferring the 3-D geoacoustic properties of the bottom, and improve our ability to localize and track sources. The experimental configuration consists of a source radiating one or more pure tones to a field of freely drifting buoys, each containing a hydrophone, GPS navigation, and radio telemetry. A key component of the method is the establishment of a local differential GPS system between the source ship and each buoy, thereby enabling the determination of the positions of the buoys relative to the ship with submeter accuracy. In this manner, the drifting buoys create 2-D synthetic aperture horizontal arrays along which the modal evolution of the waveguide can be observed in the spatial domain, or after beam forming, in the horizontal wavenumber domain. Typical results from two modal mapping experiments (MOMAX) are presented in which fixed and moving source configurations were used to transmit pure tones in the band 50-300 Hz to several buoys at ranges up to 10 km. MOMAX I was conducted in about 70 m of water off the New Jersey coast in March, 1997, while MOMAX II was carried out in 50-150 m water depths in the Gulf of Mexico in February, 1999. A striking feature of these data is the remarkable stability and regularity of the phase, although the magnitude displays a complex multimodal interference pattern. A phase model is described which accurately predicts the source-receiver speed from these phase measurements. In addition, examples of modal maps are presented which illustrate the influence of lateral variations in the waveguide properties on the spatially evolving spectral content of the modal field.

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Modal Mapping in Shallow Water Using Synthetic Aperture Horizontal Arrays

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Abstract – **An experimental technique is described for mapping the wavenumber spectrum of the normal mode field as a function of position in a shallow water waveguide with three-dimensional variation in its acoustic properties. These modal maps provide a characterization of the modal properties of the waveguide, can be used as input data to inversion techniques for inferring the 3-D geoacoustic properties of the bottom, and improve our ability to localize and track sources. The experimental configuration consists of a source radiating one or more pure tones to a field of freely drifting buoys, each containing a hydrophone, GPS navigation, and radio telemetry. A key component of the method is the establishment of a local differential GPS system between the source ship and each buoy, thereby enabling the determination of the positions of the buoys relative to the ship with submeter accuracy. In this manner, the drifting buoys create 2-D synthetic aperture horizontal arrays along which the modal evolution of the waveguide can be observed in the spatial domain, or after beam forming, in the horizontal wavenumber domain. Typical results from two modal mapping experiments (MOMAX) are presented in which fixed and moving source configurations were used to transmit pure tones in the band 50-300 Hz to several buoys at ranges up to 10 km. MOMAX I was conducted in about 70 m of water off the New Jersey coast in March, 1997, while MOMAX II was carried out in 50-150 m water depths in the Gulf of Mexico in February, 1999. A striking feature of these data is the remarkable stability and regularity of the phase, although the magnitude displays a complex multimodal interference pattern. A phase model is described which accurately predicts the source-receiver speed from these phase measurements. In addition, examples of modal maps are presented which illustrate the influence of lateral variations in the waveguide properties on the spatially evolving spectral content of the modal field.**

I. Introduction

It is well known that the normal modes of an acoustic waveguide are fundamental constituents of the sound field and have properties which are intimately related to the characteristics of the waveguide and its boundaries [1]. Detailed knowledge of the modal behavior therefore provides an enormous amount of information about the propagation characteristics of the waveguide and can also be used to infer the acoustic properties of the environment. An extremely useful characterization involves the modal wavenumber spectrum obtained by beam forming CW data measured on a real or synthetic aperture horizontal array. In a horizontally stratified environment, the wavenumber spectrum has peaks at horizontal wavenumbers corresponding to the eigenvalues of the perfectly trapped modes; the heights of the peaks are proportional to the strengths of these modes. The modal eigenvalues can be used as input data to a variety of inversion schemes for inferring the geoacoustic properties of the seabed [2]. These concepts can be extended to laterally varying waveguides as well, based on the notion, originating in adiabatic mode theory, that the local modes adapt to the local environment [2,3]. Here, the modal evolution of the waveguide with range is influenced by lateral variations in the geoacoustic parameters. The variation of the modal spectrum with range can be determined by beam forming the array data over a sequence of short, sliding sub-apertures and juxtaposing the results.

In this paper, we describe a synthetic aperture methodology for implementing this modal characterization technique. We present results from two modal mapping experiments (MOMAX) that illustrate modal evolution due to lateral variations in waveguide properties. We also show that high-resolution measurements of this type can improve our ability to localize and track sources.

II. Experimental Description

The experimental configuration consists of a source radiating one or more pure tones to a field of freely drifting buoys, each equipped with a hydrophone, GPS navigation, and radio telemetry, as shown in Fig. 1. A key component of the method is the establishment of a local differential GPS

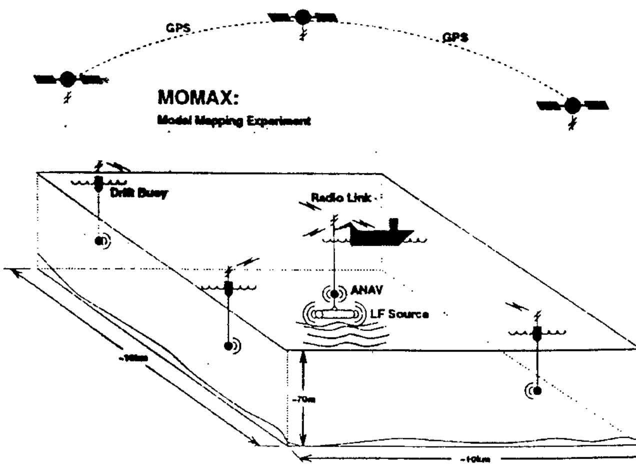

Fig. 1. Experimental configuration for the Modal Mapping Experiment.

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system between the source ship and each buoy, thereby enabling the determination of the positions of the buoys relative to the ship with submeter accuracy [4,5]. In this manner, the drifting buoys create 2-D synthetic aperture horizontal arrays along which the modal evolution of the waveguide can be observed in the spatial domain or, after beam forming, in the horizontal wavenumber domain.

The MOMAX measurements were obtained using fixed and moving source configurations transmitting pure tones in the band 50-300 Hz to several buoys at ranges up to 10 km. Both the source and the receivers remained at fixed depths (typically 30 m) during the course of the experiments. MOMAX I was conducted in about 70 m of water off the New Jersey coast in March 1997, while MOMAX II was carried out in 50-150 m water depths in the Gulf of Mexico in February 1999.

III. Experimental Results

The magnitude $|P(t)|$ and phase $\phi(t)$ of 50 Hz data from MOMAX I, after shifting the signal to base band, are shown in Figs. 2a and 2b, respectively [*i.e.*, the carrier frequency term $\exp(-i\omega t)$, where $\omega = 2\pi f$ and f = 50 Hz, has been removed]. By merging these measurements with the GPS-derived source-receiver range $r(t)$ in Fig. 2c, we can obtain a conventional display of pressure magnitude and phase versus range. However, a more interesting and novel display is shown in Fig. 3, where $|P(t)|$ and $\phi(t)$ have been merged with the actual two-dimensional positions $x(t)$ and $y(t)$ of the receiver relative to the ship (note that $r^2 = x^2 + y^2$). Thus we can observe the detailed, two-dimensional modal evolution of the acoustic field in the spatial domain. Note that in this case, both source and receiver are moving, and therefore the coordinate system in Fig. 3 is moving relative to fixed earth coordinates.

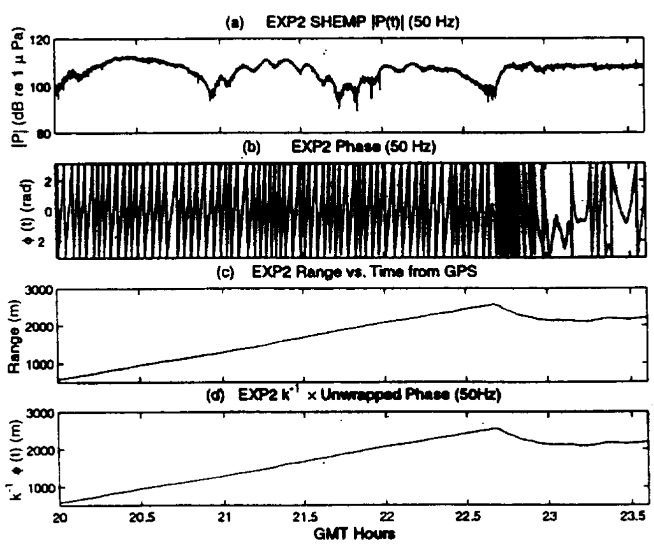

Fig. 2. MOMAX I – 50 Hz data versus time: (a) pressure magnitude, (b) pressure phase, (c) GPS source-receiver range, (d) k^{-1} x unwrapped phase.

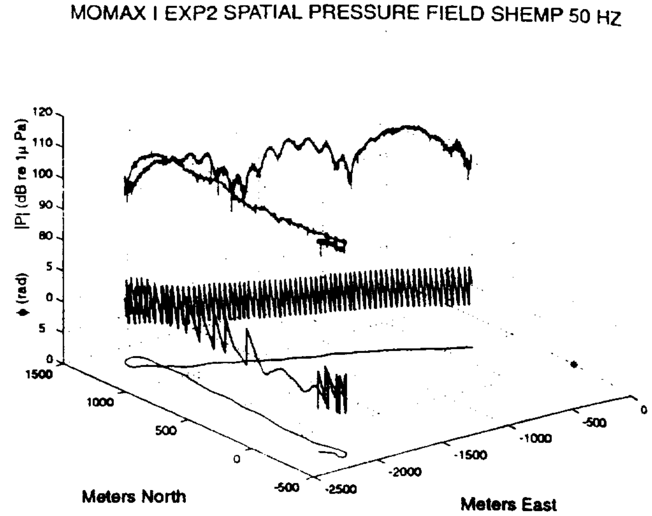

Fig. 3. MOMAX I – 50 Hz pressure magnitude and phase versus 2-D receiver position. Asterisk at lower right indicates source position and lowest line indicates receiver trajectory.

A. Phase Rate and Source-Receiver Motion

A striking feature of the data in Figs. 2 and 3 is the remarkable stability and regularity of the phase, although the magnitude displays a complex multimodal interference pattern. It is also apparent that the phase rate seems to be directly correlated with the range rate between source and receiver. In fact, a phase model has been developed [6] which accurately describes this behavior:

$$\frac{dr}{dt} = \frac{1}{k}\frac{d\phi}{dt} + \dots, \qquad (1)$$

where $k = \omega/c$ and c is a characteristic sound speed in the water column (in this case, c = 1490 m/s). When this model is applied to the 50 Hz data, we obtain the excellent comparison between $k^{-1}\,d\phi/dt$ and the GPS-derived dr/dt shown in Fig. 4. Thus the source-receiver range rate can be inferred from measurements of the low-frequency acoustic phase rate. Reference 6 indicates that this phase model is applicable to much longer range, deep water, range-dependent conditions as well.

When we integrate both sides of Eq. (1) with respect to time, we obtain

$$r = \frac{\phi}{k} + K, \qquad (2)$$

where K is an unknown integration constant. Equation (2 indicates that we can infer the source-receiver range from th measured, unwrapped phase if we specify the starting rang (corresponding to K). By substituting the initial GPS rang measurement at 20 hr GMT and the 50 Hz unwrapped phas data into Eq. (2), we obtain inferred ranges in Fig. 2d whic

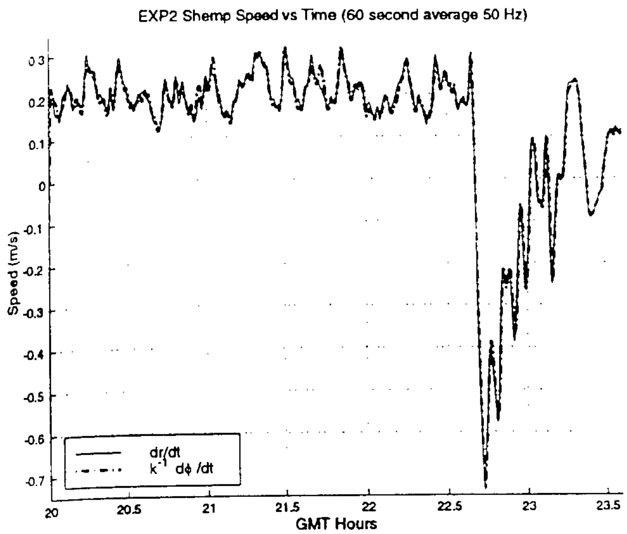

Fig. 4. Comparison of GPS-derived phase rate (solid line) with phase model results (dashed line) obtained using MOMAX I – 50 Hz data.

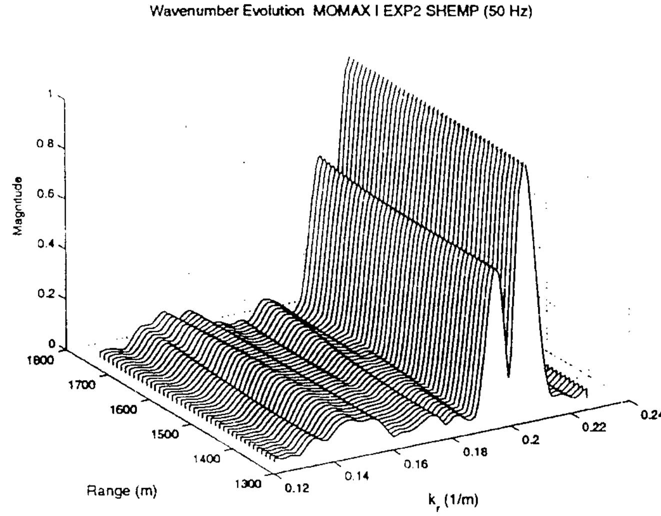

Fig. 5. Modal evolution for the MOMAX I – 50 Hz data.

show excellent agreement with the GPS-derived ranges shown in Fig. 2c.

B. Modal Evolution in the Horizontal Wavenumber Domain

In the case of a laterally varying waveguide, where both the sound speed in the water column and the properties of the bounding media are assumed to be slowly varying functions of range, we can express the acoustic field p in terms of the adiabatic mode sum [1]

$$p(r,z) = A\sum_n u_n(0,z_0)u_n(r,z)\frac{\exp\left[i\int_0^r k_n(r')dr'\right]}{\sqrt{k_n(r)r}}, \tag{3}$$

where $A=\sqrt{2\pi}\exp(i\pi/4)/\rho$, ρ is the constant density in the water column, z_0 and z are the source and receiver depths, respectively, and the eigenfunctions $u_n(r,z)$ and eigenvalues $k_n(r)$ satisfy a local eigenvalue equation and local boundary conditions at range r. Suppose that the modal field is measured on a horizontal array of length L, and further assume that $u_n(r,z)\approx u_n(R,z)$ and $k_n(r)\approx k_n(R)$ over the interval $R\le r\le R+L$. Then the beam response $\mathrm{P}(k_r)$ for this array can be determined by computing the Fourier transform [2]

$$\mathrm{P}(k_r) = \frac{1}{\sqrt{2\pi}}\int_R^{R+L}\sqrt{r}\,p(r)e^{-ik_r r}dr, \tag{4}$$

where k_r is the horizontal wavenumber. Substituting Eq. (3) into Eq. (4), we obtain

$$\mathrm{P}(k_r,R,z) = \sum_n B_n u_n(0,z_0)u_n(R,z)\frac{\sin\{[k_r-k_n(R)]L/2\}}{[k_r-k_n(R)]}, \tag{5a}$$

$$B_n = \sqrt{2/\pi\, k_n(R)}\,A e^{i\left\{\int_0^R k_n(r')dr' - k_n(R)R - [k_r-k_n(R)](R+L/2)\right\}}. \tag{5b}$$

It is clear from Eq. (5) that there are peaks in the beam response at the horizontal wavenumbers $k_r = k_n(R)$ corresponding to the modal eigenvalues at range R. Furthermore, we see from Eq. (5) that the range evolution of the modal peak positions $k_n(R)$ can be inferred from the horizontal array measurements by applying the beam-forming operation over a succession of sub-apertures in range.

An example of the application of this approach to the MOMAX I – 50 Hz data is shown in Fig. 5. In this case, the data were beam formed over a succession of 1600 m sub-apertures, where the position of each sub-aperture was shifted from the previous one by 10 m. It is clear that there are two dominant, stable modes in this relatively benign waveguide environment. They correspond to the wavenumber manifestation of the predominantly bimodal interference pattern we observed in the spatial domain in Fig. 3.

In a second example, we apply this technique to 250 Hz data obtained in MOMAX II. Using a 2000 m aperture slid in 40 m steps, we obtain the results shown in Fig. 6. In this sloping bottom environment, we can see the evolution of a rich modal structure with range. In particular, we observe how the two dominant modes that exist at a range of 1000 m gradually merge into a single mode at 1700 m.

IV. Conclusions

We have described a synthetic aperture technique for mapping the wavenumber spectrum of the normal mode field as a function of position in a shallow water waveguide with three-dimensional variation in its acoustic properties. The experimental configuration consists of a source transmitting

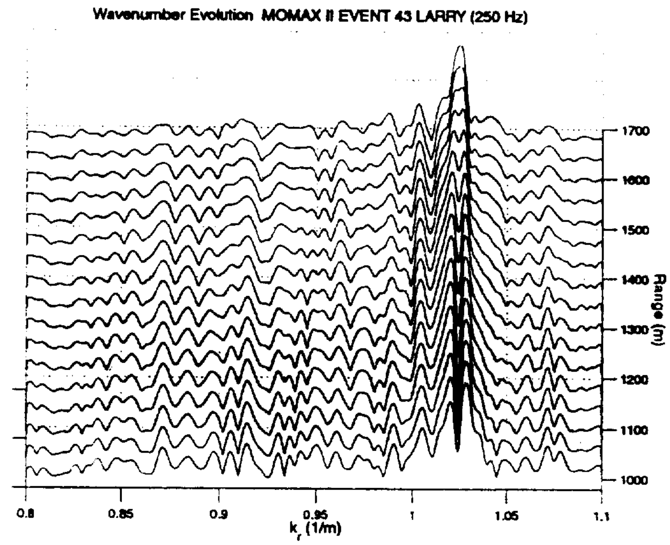

Fig. 6. Modal evolution for the MOMAX II – 250 Hz data.

one or more pure tones to a field of freely drifting buoys, each equipped with a hydrophone, GPS navigation, and radio telemetry. Typical results from two modal mapping experiments at 50 Hz and 250 Hz, illustrating modal evolution in the spatial and horizontal wavenumber domains, were presented. In addition, a phase model was described which accurately predicts the source-receiver range rate from measurements of the low-frequency acoustic phase rate.

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References

[1] G. V. Frisk, *Ocean and Seabed Acoustics: A Theory of Wave Propagation*, Prentice-Hall, Englewood Cliffs, NJ, 1994.

[2] G.V. Frisk, "A review of modal inversion methods for inferring geoacoustic properties in shallow water," invited paper in *Full Field Inversion Methods in Ocean and Seismo-Acoustics*, O. Diachok, A. Caiti, P. Gerstoft, H. Schmidt, eds., Kluwer, Netherlands, 1995, pp. 305-310.

[3] K. Ohta and G.V. Frisk, "Modal evolution and inversion for seabed geoacoustic properties in weakly range-dependent, shallow-water waveguides," *IEEE J. Oceanic Engineering Special Issue on Shallow-Water Acoustics II*, vol. 22, pp. 501-521, 1997.

[4] J.A. Doutt, G.V. Frisk, and H. Martell, "Determination of distance between a moving ship and drifting buoys to centimeter-level accuracy at sea using L1 phase receivers and differential moving-base kinematic processing," in *Proceedings of the Institute of Navigation GPS-98 Conference*, Nashville, Tennessee, 6 pages (15-18 September 1998).

[5] J.A. Doutt, G.V. Frisk, and H. Martell, "Using GPS at sea to determine the range between a moving ship and a drifting buoy to centimeter-level accuracy," in *Proceedings of the Oceans '98 Conference*, Nice, France, 4 pages (28 September – 1 October 1998).

[6] G.V. Frisk, "Inference of source-receiver motion from phase measurements in the Modal Mapping Experiment," *J. Acoust. Soc. Am.*, vol. 107, p. 2889 (2000).